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A HIGHLY IDEALIZED MATHEMATICAL, DIFFERENTIAL STICKING CABLE MODEL TO ESTIMATE THE INFLUENCE OF FILTER CAKE THICKNESS

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1 INTRODUCTION

Tools used for various measurements in bore holes are frequently suspended on a cable whose diameter is small compared to the bore hole diameter. This study presents a model for estimating the influence of filter cake thickness on the cable axial drag force per unit of cable length. Differential sticking results when the drag force is sufficient to prevent the tool from being lowered or retrieved in the bore hole.

Differential sticking has been studied from many points of view. A comprehensive review of the studies is given in Reference 1. Many of the studies are for drill collars in bore holes and are not applicable to the analysis presented here. This report will not attempt to review the material in Reference 1.

In this report many idealizations are introduced in order to develop a mathematically tractable model. Consequently, many of the observed features of differential sticking are not predicted by the model. On the other hand, some new insight into the differential sticking problem is offered by this idealized model. Some of the idealized features of this model are,

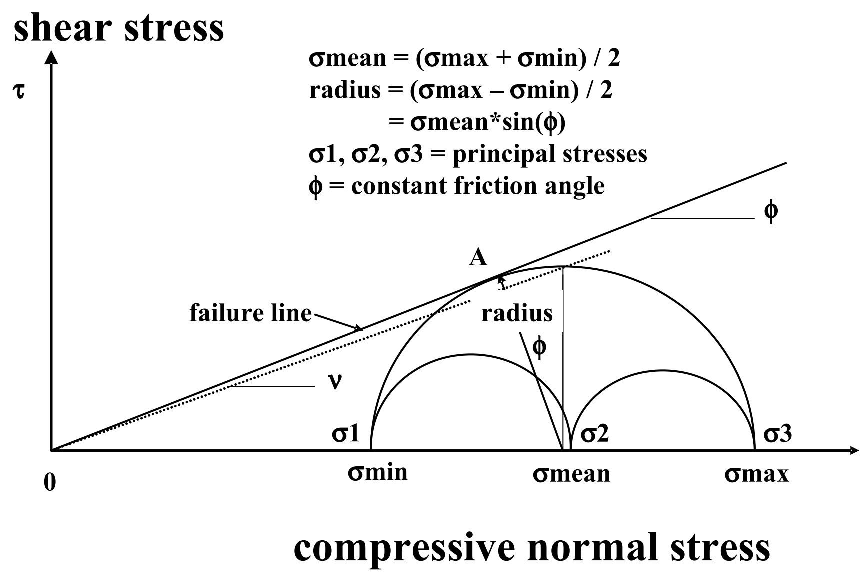
1. Owing to the diameter ratio of the cable to the bore hole being small, the borehole wall is modeled as flat with the cable resting upon and in contact with it.
2. The filter cake is modeled as a frictional (Mohr-Coulomb), porous material, see Reference 2), with a constant friction angle and a uniform permeability.
3. The filter cake material is assumed to satisfy its failure criterion everywhere. This assumption is thought to be plausible while the filter cake is building
4. The cable drag force per unit length associated with the capstan effect (= (cable tension) / (bore hole axis curvature)) is not included in the analysis. Clearly, this influence can be added to results presented here.
5. The coefficient of friction between the cable surface and the filter cake is assumed to be constant.
6. Gravity effects in the fluid and filter cake are ignored as contributions are small compared to fluid pressure influences in this study.
7. The solid particles of the filter cake are incompressible and their elastic strains are neglected.

2 NOMENCLATURE

ϕ	= friction angle for filter cake material
ζ	= (filter cake thickness) / (cable diameter)
$F\sigma V$	= force per unit length on cable toward bore hole contact by filter cake radial effective stress
$F\tau V$	= force per unit length on cable toward bore hole contact by filter cake shear stress
FpV	= force per unit length on cable toward bore hole contact by fluid pressure
$F\sigma$	= total force per unit length on cable from contact with filter cake
$DF\sigma V$	= $F\sigma V$ / $(2 \cdot r \cdot (pm - pp))$, dimensionless
$DF\tau V$	= $F\tau V$ / $(2 \cdot r \cdot (pm - pp))$, dimensionless
$DFpV$	= FpV / $(2 \cdot r \cdot (pm - pp))$, dimensionless
$DF\sigma$	= $F\sigma$ / $(2 \cdot r \cdot (pm - pp))$, dimensionless
μ_{CABLE}	= coefficient of friction between the cable and the filter
ν	= angle sometimes used to define frictional material, not used here
r	= radius of cable
α	= ccw angle from negative z-axis to cable tangent
β	= generic angle measured cw from positive z-axis
αO, βO	= angles α and β on cable cross section at top of filter cake
pm	= uniform fluid pressure inside the bore hole and filter cake
pp	= pore pressure at the bore hole wall
thk	= filter cake thickness
$p(\alpha)$	= fluid pressure on cable wall at angle α
$\sigma 1$, $\sigma 2$, $\sigma 3$	= ordered principal stresses, $\sigma 1 > \sigma 2 > \sigma 3$
σmean	= $(\sigma 1 + \sigma 3) / 2$
radius	= $(\sigma 1 - \sigma 3) / 2$
$\sigma_V(\alpha)$	= normal effective, compressive stress in z-direction
$\sigma_H(\alpha)$	= normal effective. compressive stress perpendicular to z-direction
$\sigma_N(\alpha)$	= normal effective, compressive stress from filter cake on cable wall at angle α
$\tau_N(\alpha)$	= shear stress from filter cake on cable wall at angle α
μ_{CABLE}	= coefficient of friction between the cable surface and the filter cake material

3 DESCRIPTION OF FRICTIONAL MATERIAL

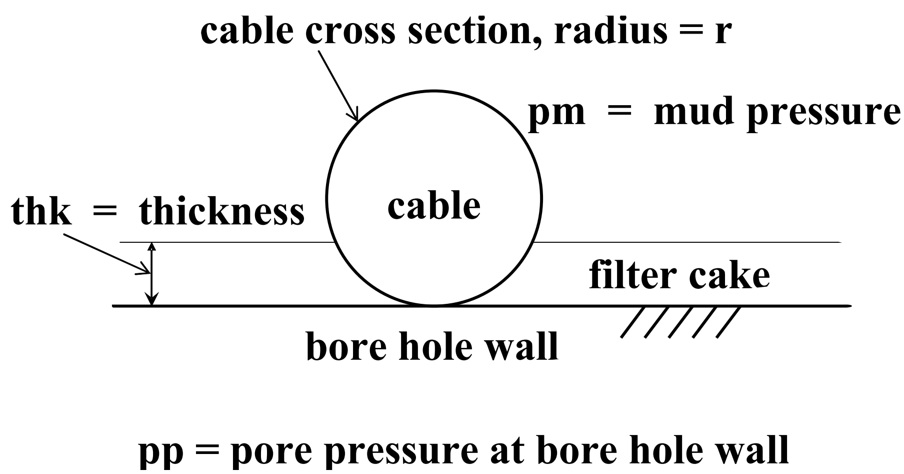
The frictional material model employed here is described in Reference 2. The total stress (actual force per unit area) is the sum of the pore pressure and the effective stress. The mechanical response of the filter cake is determined using the effective stress and is independent of the fluid pressure. A non-trivial, inelastic deformation is possible only when the material stress state satisfies the failure criterion. This criterion is met in the Mohr circle sketch below.



The failure criterion is satisfied when the radius of the largest Mohr circle contacts the failure line (point A in the sketch). Material deformation may occur when the failure condition is satisfied. The largest Mohr circle may not cross the failure line. If material deformation occurs it must be shearing on the planes determined by point A and in the direction of the shear stress at Point A on these planes.

4 ANALYSIS

The sketch below shows the configuration of the model studied in this report.



This study includes the above configuration and the case where the cable is fully buried by the filter cake (thk > $2 \cdot r$). Consequently, two mathematical derivations are required. The pressure and effective stresses in the filter cake remote from the cable have the same mathematical solution for any filter cake thickness. Let z be the coordinate measured inward from the bore hole wall and perpendicular to the bore hole wall. The pressure and effective stresses in the filter cake remote from the cable are given by,

$$p(z) = \text{fluid pressure} = pp + (pm - pp) \cdot \frac{z}{thk} \qquad 1$$

$$\sigma_V(z) = \text{normal effective stress} \perp \text{to bore hole wall} = (pm - pp) \cdot \left(1 - \frac{z}{thk}\right) \qquad 2$$

$$\sigma_H(z) = \text{normal effective stress} \parallel \text{to bore hole wall} = (1 - 2 \cdot \sin(\varphi i)) \cdot (pm - pp) \cdot \left(1 - \frac{z}{thk}\right) \qquad \mathbf{3}$$

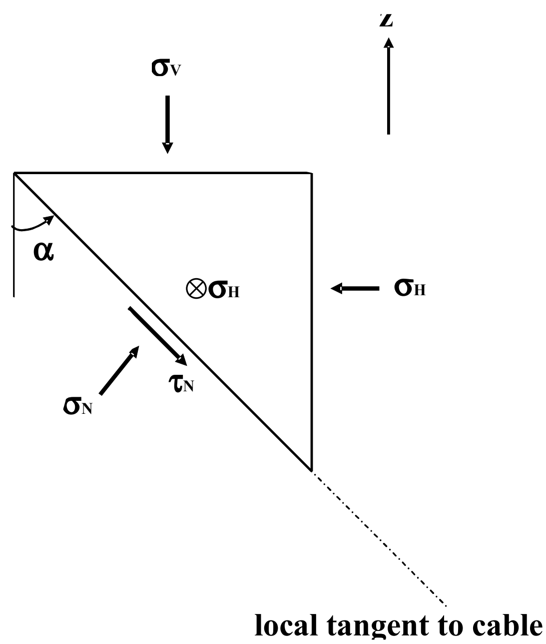
Equation 3 is derived based on the assumption that all points in the filter cake satisfy the failure criterion and that,

$$\sigma mean(z) = \text{mean normal effective stress} = (1 - \sin(\varphi i)) \cdot (pm - pp) \cdot \left(1 - \frac{z}{thk}\right) \qquad 4$$

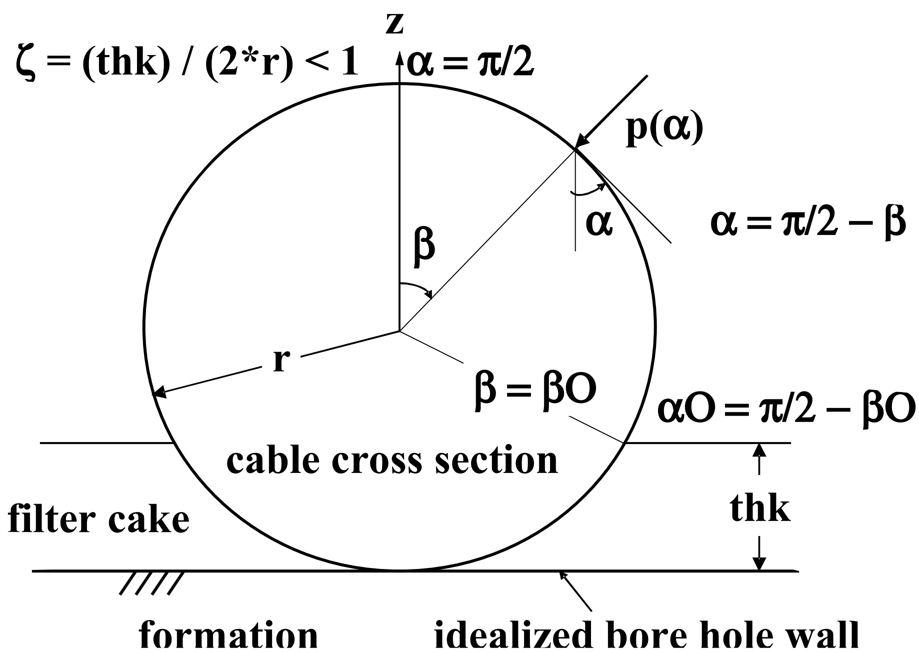
Equations 1-4 are used below to determine the loading on the cable from the filter cake. The pressure and stress distributions are assumed to be valid up to the boundary of the cable. Under this assumption, the normal effective stress and shear stress on the cable surface are shown in the figure below and.

$$\sigma_N(\alpha) = \frac{\sigma_V}{1+\sin(\phi)} \cdot (1 - \sin(\phi) \cdot \cos(2 \cdot \alpha)) \tag{5}$$

$$\tau_N(\alpha) = \frac{\sigma_V}{1+\sin(\phi)} \cdot \sin(\phi) \cdot \sin(2 \cdot \alpha) \tag{6}$$



4.1 FILTER CAKE THICKNESS < CABLE DIAMETER



The sketch above shows the parameters z (measured from the idealized bore hole wall), thk, α, β, αO, βO and ζ used for this part of the analysis. The pressure and pertinent effective stresses in $\alpha O > \alpha > -\pi/2$ are,

$$p(\alpha) = pp + (pm - pp)\cdot\frac{1+\sin(\alpha)}{1+\sin(\alpha O)} \qquad 7$$

$$\sigma_N(\alpha) = (pm - pp)\cdot\frac{\sin(\alpha O)-\sin(\alpha)}{(1+\sin(\phi))\cdot(1+\sin(\alpha O))}\cdot(1-\sin(\phi)\cdot\cos(2\cdot\alpha)) \qquad 8$$

$$\tau_N(\alpha) = (pm - pp)\cdot\frac{\sin(\alpha O)-\sin(\alpha)}{(1+\sin(\phi))\cdot(1+\sin(\alpha O))}\cdot\sin(\phi)\cdot\sin(2\cdot\alpha) \qquad 9$$

and

$$\zeta = 0.5\cdot[1+\cos(\alpha O - 0.5\cdot\pi)] = 0.5\cdot[1+\sin(\alpha O)] \qquad 10$$

The above pressure and effective stresses are used to evaluate the following dimensionless forces for $thk \le 2\cdot r$.

$$DF\sigma V = F\sigma V\ /\ (2\cdot r\cdot(pm-pp)) = \int_{\alpha O}^{-\pi/2}\frac{\sigma_N(\alpha)\cdot\sin(\alpha)}{pm-pp}\cdot d\alpha$$

$$= \begin{Bmatrix} -\sin(\alpha O)\cdot\cos(\alpha O) - 0.5\cdot(\alpha O + 0.5\cdot\pi) + 0.25\cdot\sin(2\cdot\alpha O) \\ -\sin(\phi)\cdot\sin(\alpha O)\cdot\cos(\alpha O)\cdot\left[1 - 2\cdot\cos^2(aO)/3\right] \\ +\sin(\phi)\cdot\left[-0.25\cdot(\alpha O + 0.5\cdot\pi) + 0.125\cdot\sin(2\cdot\alpha O) + 0.5\cdot\sin^3(\alpha O)\cdot\cos(\alpha O)\right] \end{Bmatrix} / \left[(1+\sin(\phi))\cdot(1+\sin(\alpha O))\right] \tag{11}$$

$$DF\tau V = F\tau V\,/\,(2\cdot r\cdot(pm-pp)) = \int_{\alpha O}^{-\pi/2} \frac{\tau_N(\alpha)\cdot\cos(\alpha)}{pm-pp}\cdot d\alpha$$

$$= \begin{matrix} -\{\sin(\phi)/[(1+\sin(\phi))\cdot(1+\sin(\alpha O))]\} \\ \cdot\left[2\cdot\sin(\alpha O)\cdot\cos^3(\alpha O)/3 + \pi/8 + \alpha O/4 - \sin(4\cdot\alpha O)/16\right] \end{matrix} \tag{12}$$

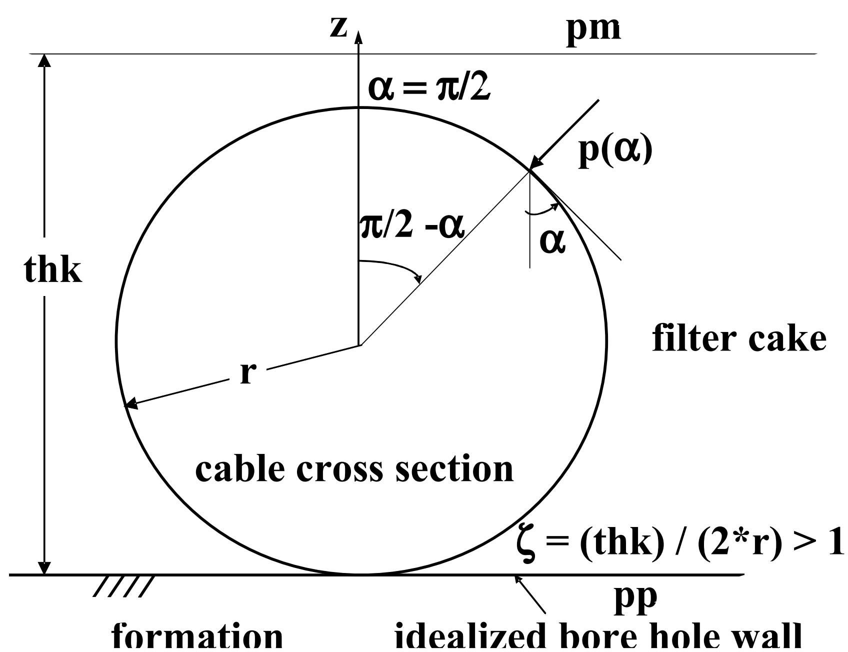
$$DFpV = FpV\,/\,(2\cdot r\cdot(pm-pp)) = \int_{\alpha O}^{-\pi/2} \frac{p(\alpha)\cdot\sin(\alpha)}{pm-pp}\cdot d\alpha$$

$$= \cos(\alpha O) + [-\cos(\alpha O) + 0.5\cdot\alpha O + 0.25\cdot\pi - 0.25\cdot\sin(2\cdot\alpha O)]/[1+\sin(\alpha O)] \tag{13}$$

$$DF\sigma = F\sigma\,/\,(2\cdot r\cdot(pm-pp)) = \int_{\alpha O}^{-\pi/2} \frac{\sigma_N(\alpha)}{pm-pp}\cdot d\alpha$$

$$= \begin{Bmatrix} [1-\sin(\phi)]\cdot[(0.5\cdot\pi+\alpha O)\cdot\sin(\alpha O)\cdot\cos(\alpha O)] \\ +2\cdot\sin(\phi)\cdot\left[\sin(\alpha O)\cdot\begin{pmatrix} 0.25\cdot\pi+0.5\cdot\alpha O \\ -0.25\cdot\sin(2\cdot\alpha O)\end{pmatrix} + \cos(\alpha O)\cdot(1-\cos^2(\alpha O)/3)\right] \end{Bmatrix} / \left[(1+\sin(\phi))\cdot(1+\sin(\alpha O))\right] \tag{14}$$

4.2 FILTER CAKE THICKNESS > CABLE DIAMETER



The sketch above shows the parameters z (measured from the idealized bore hole wall), thk, α, αO and ζ used for this part of the analysis. The pressure and pertinent effective stresses in π/2 > α > -π/2 are,

$$p(\alpha) = pp + \frac{1}{2\cdot\zeta}\cdot(1+\sin(\alpha))\cdot(pm-pp) \quad 15$$

$$\sigma_N(\alpha) = (pm-pp)\cdot\frac{1-\sin(\phi)\cdot\cos(2\cdot\alpha)}{1+\sin(\phi)}\cdot\left(1-\frac{1}{2\cdot\zeta}(1+\sin(\alpha))\right) \quad 16$$

$$\tau_N(\alpha) = (pm-pp)\cdot\frac{\sin(\phi)\cdot\sin(2\cdot\alpha)}{1+\sin(\phi)}\cdot\left(1-\frac{1}{2\cdot\zeta}(1+\sin(\alpha))\right) \quad 17$$

The above pressure and effective stresses are used to evaluate the following dimensionless forces for thk $\geq 2\cdot r$.

$$DF\sigma V = F\sigma V\,/\,(2\cdot r\cdot(pm-pp)) = \int_{\pi/2}^{-\pi/2}\frac{\sigma_N(\alpha)\cdot\sin(\alpha)}{pm-pp}\cdot d\alpha$$

$$= [-0.25\cdot\pi\cdot(1/\zeta)\cdot(1+0.5\cdot\sin(\phi))]\,/\,[1+\sin(\phi)] \quad 18$$

$$DF\tau V = F\tau V\,/\,(2\cdot r\cdot(pm-pp)) = \int_{\pi/2}^{-\pi/2}\frac{\tau_N(\alpha)\cdot\cos(\alpha)}{pm-pp}\cdot d\alpha$$

$$= -[\sin(\phi)/(1+\sin(\phi))]\cdot 0.125\cdot\pi\cdot(1/\zeta) \quad 19$$

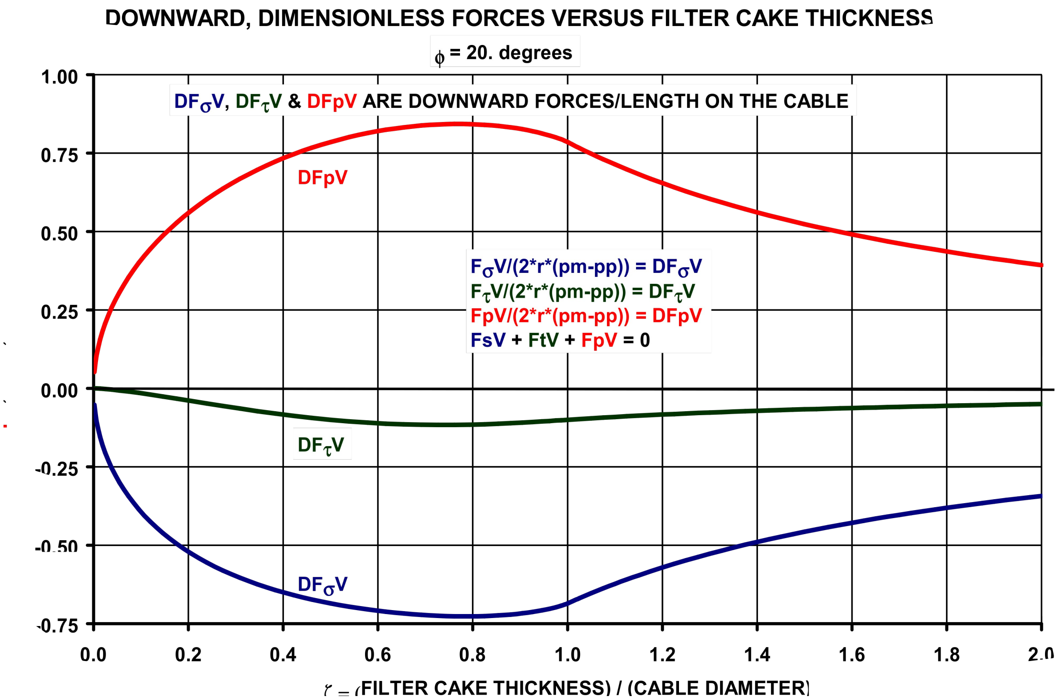
$$\mathrm{DFpV} = \mathrm{FpV} \,/\, \left(2 \cdot \mathrm{r} \cdot (\mathrm{pm} - \mathrm{pp})\right) = \int_{\pi/2}^{-\pi/2} \frac{\mathrm{p}(\alpha) \cdot \sin(\alpha)}{\mathrm{pm} - \mathrm{pp}} \cdot \mathrm{d}\alpha$$

$$= 0.25 \cdot \pi \cdot (1/\zeta) \qquad 20$$

$$\mathrm{DF}\sigma = \mathrm{F}\sigma \,/\, \left(2 \cdot \mathrm{r} \cdot (\mathrm{pm} - \mathrm{pp})\right) = \int_{\pi/2}^{-\pi/2} \frac{\sigma_{\mathrm{N}}(\alpha)}{\mathrm{pm} - \mathrm{pp}} \cdot \mathrm{d}\alpha$$

$$= \pi \cdot \left[1 - 0.5 \cdot (1/\zeta)\right] / \left(1 + \sin(\phi)\right) \qquad 21$$

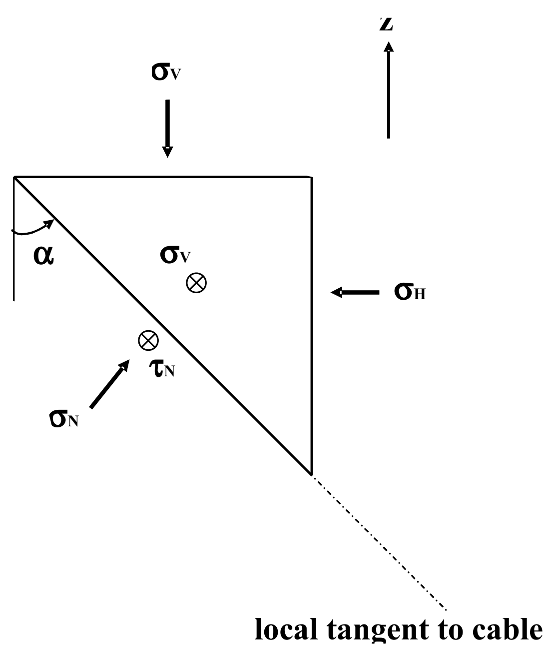
SOME GRAPHICAL RESULTS



The forces per unit length acting on the cable must vanish in order to satisfy the force equilibrium conditions. Owing to the symmetry of the model, the forces perpendicular to the cable axis and parallel to the idealized bore hole wall all vanish. In addition, symmetry ensures that that there is no moment per unit length about the cable axis. The above figure gives the vertical contributions to the force on the cable from pressure and wall stresses. It may be verified that the sum of $F\sigma V$, $F\tau V$ and FpV is zero so the solution considered here satisfies all equilibrium conditions.

CRUDE ESTIMATE OF CABLE DRAG FORCE PER UNIT LENGTH

The stress states depicted in Equations 7, 8 & 9 on Page 6 and Equations 15, 16 & 17 on Page 8 are in the failure state and, by assumption the material continues to be in the failure state. The vertical, compressive, effective stress is greater than the other two principal stresses which are equal to one another. When an axial load is applied to the cable attempting to slide the cable relative to the filter cake, a shear stress parallel to the cable axis on the inclined face on Page 6 is required to resist potential sliding. This may be accomplished by changing only the principal effective stress parallel to the cable axis. It must be changed to equal the vertical, compressive, effective stress. This change may be visualized using the sketch on Page 3. Before the axial load is applied, $\sigma 1 = \sigma 2 < \sigma 3$ while application of the axial load moves $\sigma 2$ to equal $\sigma 3$ so that the failure criterion is reached when $\sigma 1 < \sigma 2 = \sigma 3$. This change implies a possible discontinuity in the circumferential stress that is acceptable for the ridgid-frictional material model. The result of the new stress state is that the vector labeled τ_N on Page 5 now is perpendicular to the sketch as shown below.



The effective normal and shear stresses on the cable surface for the new stress state have the same magnitudes as those on Page 5. That is,

$$\sigma_N(\alpha) = \frac{\sigma_V}{1+\sin(\phi)} \cdot (1 - \sin(\phi)\cdot\cos(2\cdot\alpha)) \qquad 22$$

$$\tau_N(\alpha) = \frac{\sigma_V}{1+\sin(\phi)} \cdot \sin(\phi)\cdot\sin(2\cdot\alpha) \qquad 23$$

This is not a possible state for cable sliding based on shearing in the material according to the frictional material formulation. In general, the boundary stresses from equations 22 and 23 do not correspond to Point A indicated in the sketch on Page 3. The direction of sliding must be axial so that all points on the cable boundary must have $\sigma_N(\alpha)$ and $\tau_N(\alpha)$ values at Point A. This condition requires that $\frac{\tau(\alpha)}{\sigma(\alpha)} = \tan(\phi)$ or, using Equations 22 and 23 and a little trigonometry,

$$\phi + 2 \cdot \alpha = \frac{\pi}{2} \qquad 24$$

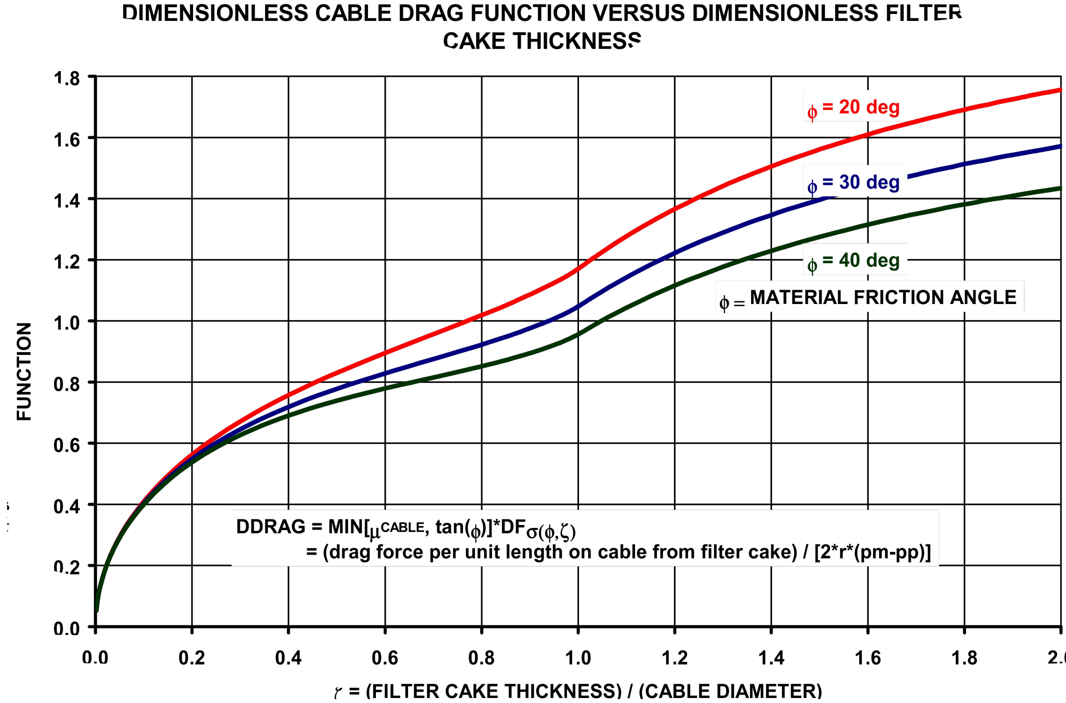
Since α varies with position on the cable surface and ϕ is a constant, Equation 24 cannot be satisfied by the stress state given by Equations 23 and 24. Clearly, the solution to this problem is more complex than the simple stress fields considered here.

The assumption that the filter cake meets the failure criterion everywhere and that during cable axial sliding the size of the region undergoing shearing around the cable has a small radial thickness can be used to make a crude approximation to the axial load per unit cable length required to initiate sliding. The assumption concerning a small radial thickness is based on the solution of the equilibrium equations with uniform pressure and all stresses dependent on radial position only. In this case σ_{rz} varies inversely with r. This approximation uses the product of the radial stress at the cable surface from Equation 22 and $\tan(\phi)$ to estimate the axial shear stress on the cable surface at incipient slipping. When this is integrated over the cable surface the resulting force is found for having the frictional material undergo shearing.

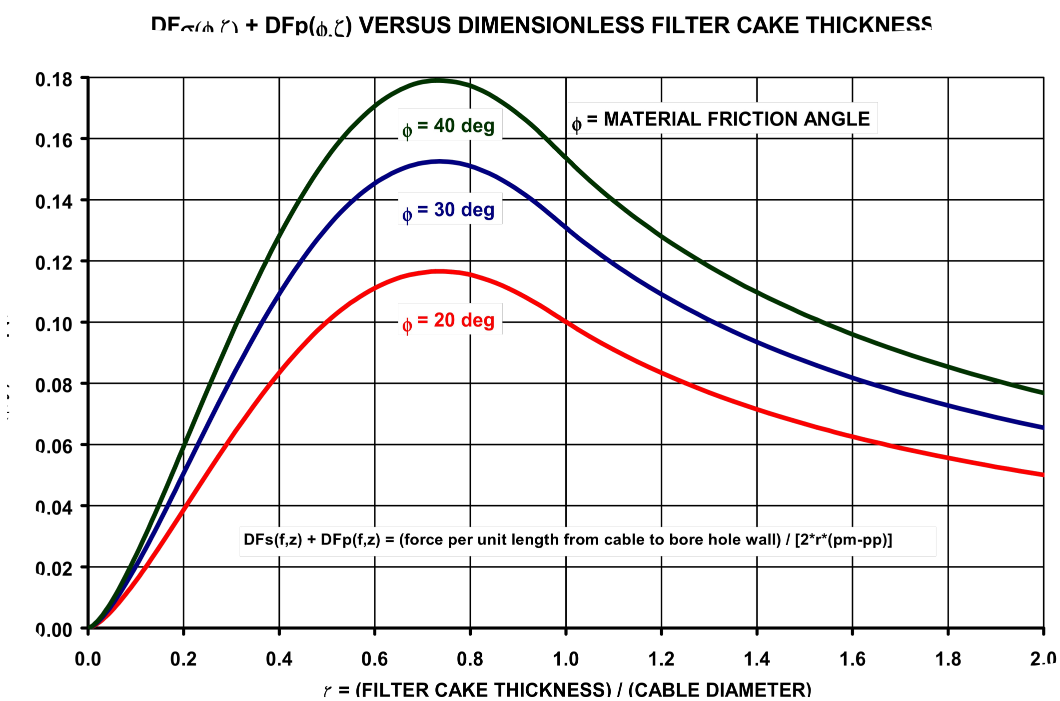
The crude approximation described in the preceding paragraph includes an implicit assumption that the axial load on the cable can be reached before slippage occurs. The possibility that the coefficient of friction between the cable and the filter cake, μ_{CABLE}, is not high enough to prevent cable slippage relative to the filter cake before the crude approximation load is reached must be recognized. This possibility may be taken into account by the following formulation.

$$\begin{aligned} \mathrm{DDRAG} &= \frac{\text{(axial cable drag force per unit length to initiate sliding)}}{2 \cdot r \cdot (pm - pp)} \\ &= \mathrm{MIN}(\mu_{CABLE}, \tan(\phi)) \cdot \mathrm{DF}\sigma \end{aligned} \qquad 25$$

where $\mathrm{MIN}(\mu_{CABLE}, \tan(\phi))$ equals the minimum value of μ_{CABLE} and $\tan(\phi)$. The plot below shows the dependence of $\mathrm{DF}\sigma$ on ϕ.



Finally, the earlier result that the sum of FσV, FτV and FpV equals zero is no longer valid as the direction of the cable wall shear stress has changed direction. There is a force per unit length of FσV + FpV acting on the cable normal to its axis. The plot below gives this force per unit length in dimensionless form.



The above figure can be used to find the lateral load per unit length on the cable from the bore hole. This load should be added to the capstan load effect (= (cable tension) / (bore hole axis curvature)) to obtain the total lateral load per unit length between the cable and the bore hole. This load should be multiplied by the coefficient of friction between the cable and the bore hole and the result added to the drag load obtained using the top plot on Page 13.

REFERENCES

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